CHAPTER 8: ENGINEERING ESTIMATIONS AND APPROXIMATIONS

In this lesson we will cover:

- Significant Digits
- Accuracy and Precision
- Errors
- Approximations

1.0 INTRODUCTION

Engineers make measurements of a vast array of physical quantities that control the design solution. Skill in making and interpreting measurements is essential element of our practice in engineering.

2.0 SIGNIFICANT DIGITS

Any physical measurements cannot be assumed to be exact. Errors are likely to be present regardless of the precautions used when making the measurement. Quantities determined by the analytical means are not always exact either. Often assumptions are made to arrive at an analytical expression, which is then used to calculate a numerical value.

It is clear that a method of expressing results and measurements is needed that will convey how "good" these numbers are. The use of ***significant digits*** gives us this capability without resorting to the more rigorous approach of computing an estimated percentage error to be specified with each numerical result or measurement.

A ***significant digit***, or ***figure,*** is defined as any digit used in writing number except those zero that are used only for location of the decimal point or those zeros that do not have any nonzero digit on their left. When you read the number 0.0015, only the digits 1 and 5 are significant since the zeros have no nonzero digit to their left. We would say then that this number has two significant figures. If the number is written 0.00150, it contains three significant figures; the rightmost zero is significant.

Numbers 10 or larger that are not written in scientific notation and that are not counts (exact values) can cause difficulties in interpretation when zeros are present. For example, 2 000 could contain one, two, three, or four significant digits – it is not clear which. To make it clear, write the number in scientific notation: 2.0×10^3 – this number has two significant digits.

When reading instruments, such as an engineer's scale, thermometer or fuel gauge, the last digit will normally be an estimate. That is, the instrument is read by estimating between the smallest graduations on the scale to get the final digit.

Quantity	Number of Significant Figures
4784	4
36	2
60	1 or 2
600	1,2, or 3
6.00×10^2	3
31.72	4
30.02	4
46.0	3
0.02	1
0.020	2
600.00	5

As you perform arithmetic operations, it is important that you not lose the significance of your measurements or, conversely, imply precision that does not exist. Rules for determining the number of significant figures that should be reported have been developed. They are as follows:

Multiplication and Division

The product or quotient should contain the same number of significant digits as are contained in the number with the fewest significant digits.

Addition and Subtraction

The answer should show significant digits only as far to the right as is seen in the least precise number in the calculation.

Combined Operations

If products are to be added or subtracted, perform the multiplication or division first, establish the correct number of significant figures in the subanswer, then perform the addition of subtraction, and round to proper significant figures.

3.0 ACCURACY AND PRECISION

In measurements accuracy and precision have different meanings and cannot be used interchangeably. ***Accuracy*** is the measure of the nearness of a value to the correct or true value. ***Precision*** refers to the repeatability of a measurement, that is, how close successive measurements are to each other. Figure 1 illustrates accuracy and precision of the results of four dart throwers.

- Thrower (a) is both inaccurate and imprecise because the results are away from the bull's eye (accuracy) and widely scattered (precision).
- Thrower (b) is accurate because the throws are evenly distributed about the desired result but imprecise because of the wide scatter.
- Thrower (c) is precise with tight cluster of throws but inaccurate because the results are away from the bull's eye.
- Finally thrower (d) demonstrates accuracy and precision with a tight cluster of throws around the center of the target.
- Thrower (a), (b), and (c) can improve their performance by analyzing the causes for the errors. Body position, arm and release point could deviation from the desired result.

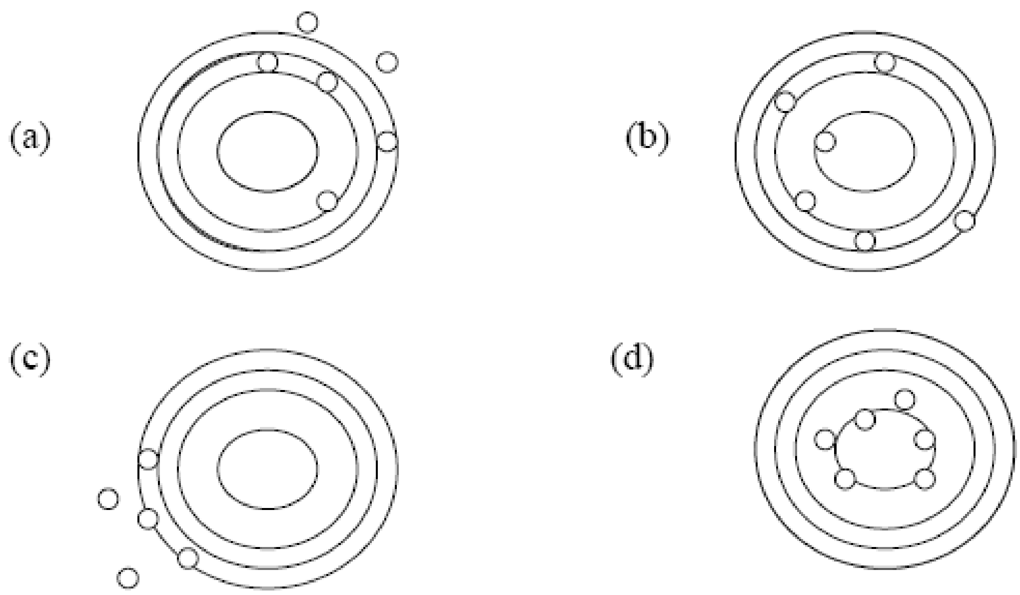

Figure 1 Illustration of the difference between accuracy and precision in physical measurements.

Engineers making physical measurements encounter two types of errors; systematic and random. These will be discussed in the next section.

Measurements can be represented as a value plus or minus (±) a number, for example, 32.3± 0.2. This indicates a range of values that are equally representative of the indicated value (32.3). Thus 32.3, 32.1 and 32.5 are among the 'acceptable' values for this measurement, A range of permissible error also can be specified as a percentage of the indicated value. For example, a thermometer's accuracy may be specified as ± 1.0

percent full-scale reading. Thus if the full scale reading is 220°F, reading should be within ±2.2 of the true value (220 x 0.01 = 2.2)

4.0 ERRORS

To measure is to err! Anytime a measurement is taken, the result is being compared to a true value, which itself may not be known exactly. If we measure the dimension of a room, why does a repeat of the dimension not yield the same result? Did the same person make all the measurements? Was the same measuring instruments used? Were the readings all made from the exactly the same eye position? Was the measuring instrument correctly graduated?

It is obvious that errors will occur in each measurement. We must try to identify the error, if we can, and correct them in our results. If we cannot identify the error, we must provide some conclusions as to the resulting accuracy and precision of our measurements.

Identifiable and correctable errors are classified as systematic; accidental or other non-identifiable errors are classified as random.

4.1 SYSTEMATIC ERRORS

Our task is to measure the distance between two fixed points. Assume that the distance is about 1200m and that we are experienced and competent and have equipment of high quality to do the measurement. Some of the errors that occur will always have the same sign (+ or -) and are said to be systematic. Assume that a 25m steel tape is to be used, one that had been compared with the standard at the U.S Bureau of Standards in Washington, DC. If the tape is not exactly 25.000m long, then they will be a systematic error each of the 48 times that we used the tape to measure out the 1200m.

4.2 RANDOM ERRORS

In reading the previous section, you may have realized that even if it had been possible to eliminate all the systematic errors, the measurement is still not exact. To elaborate on this point, we will continue with the example of the task of measuring the 1200m distance. Several random errors can creep in, as follows. When reading the thermometer, we estimate the reading when the indicator falls between graduations. Furthermore, the thermometer may be accurately measuring the tape temperature but may be influenced instead, by the temperature of the ambient air. These errors thus can produce measurements that are either too large or too small. Regarding sign and magnitude the error therefore is random.

5.0 APPROXIMATIONS

Engineers strive for a high level of precision in their work. However, it is also important to be aware of an expected precision and the time and cost of attaining it. There are many instances where an engineer is expected to make an approximation to an answer, that is, to estimate the result with reasonable accuracy but under tight time and cost constraints. To do this, engineers rely on their basic understanding of the problem under discussion coupled with their previous experience. This knowledge and experience is what distinguishes an 'approximation' from a 'guess'. If greater accuracy is needed, the initial approximation can be refined when time and funds are available and the necessary additional data for refining the results are available.

EXAMPLE:

A civil engineer is asked to meet with a city council committee to discuss their needs with respect to the disposal solid waste (garbage of refuse). The community, a city of 12000 persons, must begin supplying refuse collection and disposal for its citizen for the first time. In reviewing various alternatives for disposal for disposal, a sanitary landfill is suggested. One of the council members is concern about how much land is going to be needed in, so she asks the engineer how many acres will be required within the next 10 years.

DISCUSSION:

The engineer quickly estimates as follows:
The national average solid waste production is 2.75kg/capita/day. We can determine that each citizen thus will produce about 1000kg of refuse per year by the following calculation:

$$(2.75\text{kg/day})(365/\text{days/year}) \approx 1000\text{kg/year}$$

Experience indicates that refuse can probably be compacted to a density of 400 to 600kg/m^3. On the basic the per capita landfill volume will be about 2m^2 each year; and 1 acre filled 1m deep will contain the collected refuse of 2000 people for a year (1 acre = 4047m^2). Therefore the requirement for 12 000 people will be 1acre filled 6m deep. However, knowledge of the geology of the particular area indicates that some growth in population and solid waste generation should be expected. It is finally suggested that the city should plan to use about 20 acres in the next 10 years.

This calculation took only minutes and required no computational device other than pencil and paper. The engineer's experience, rapid calculation, sound basic assumption, and sensible rounding of figures were the main requirements. A usable estimate, designed neither to mislead not to sell a point of view, was provided. If this project proceeds to the actual development of the sanitary landfill, the civil engineer will then gather actual data, refine the calculations, and prepare estimates upon which one would risk a professional reputation.